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Improved classical theory of collisional absorption

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Abstract
We re-investigate some classical approaches for collisional absorption of laser radiation in dense plasmas and compare them to quantum theories. The typical break-down of the classical approaches can be avoided by using the quantum dielectric function in the seminal Dawson and Oberman formula which is equivalent to recently published quantum theories of collisional absorption. Strong electron–ion scattering can however be included more easily in classical approaches.

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1. Introduction

The goal of inertial confinement fusion and the understanding of large astrophysical objects give rise to renewed interest in dense plasmas and warm dense matter. Key features of such matter are strong ion–ion coupling and degenerate electrons with $\Gamma_{ii} = (Z^2e^2/k_BT_e)(4\pi n_i/3)^{1/3} > 1$ and $n_e(2\pi\hbar^2/m_ek_BT_e)^{3/2} > 1$, respectively [1, 2]. One approach to create such plasmas in the laboratory is laser–solids interactions. In particular, research on inertial fusion needs relatively long heating with intermediate laser intensity leaving the target in the warm dense matter state [3].

Laser energy can be coupled to matter by different mechanisms. In dense plasmas, inverse bremsstrahlung is the primary one for low and intermediate intensities. The first calculations of this process were done in the high frequency and small field limit [4, 5]. Later Decker *et al* generalized this approach to strong laser fields [6]. As the ballistic model of [7], these theories use classical electron–ion interactions and, thus, include an ad hoc cutoff to avoid a divergence at close collisions. Accordingly, they break down for moderately and strongly coupled plasmas.

The first quantum treatment was presented by Perel' and Éliashberg [8]. Later full quantum approaches were used to include the effects of strong laser fields during weak collisions, strong ion–ion correlations (described by an HNC approach) and strong collisions in weak laser fields using a T-matrix approach [9–14]. For certain regions, these quantum results were shown to

match molecular dynamics (MD) simulations [12, 15]. Furthermore, strong ionic correlations were included in the memory function formalism [16].

In this contribution, we use the fact that the early Dawson and Oberman approach [4] produces results identical to quantum theories [10, 11], provided that the full quantum dielectric function is used rather than its classical or small argument versions. It is also in good agreement with MD simulations [12, 15] although neither field contributions during scattering nor strong interactions are accounted for. It is shown that improved choices for the impact parameters in the classical expressions yield rather good results up to intermediate coupling strengths of $\Gamma \sim 1$.

2. Classical approaches to collisional absorption

First, we briefly review the early results for laser absorption by inverse bremsstrahlung. These were usually cast in terms of the cycle averaged collision frequency defined as $\nu_{ei} = \left(4\pi\omega_0^2/\omega_p^2\right)\langle \boldsymbol{j}\cdot\boldsymbol{E}\rangle/\langle \boldsymbol{E}\cdot\boldsymbol{E}\rangle$ by Silin [5]. Within a dielectric approach, Decker *et al* obtained for this quantity in the high-frequency limit [6]

$$\frac{\nu_{ei}}{\omega_p} = \frac{Ze^2\pi^2\omega_0}{2m_e v_0^2\omega_p}\sum_{n=-\infty}^{\infty}\int \mathrm{d}^3\boldsymbol{k}\frac{\mathrm{i}}{k^2}\frac{nJ_n^2(\boldsymbol{k}\cdot\boldsymbol{\epsilon})}{\varepsilon(k,n\omega_0)}S_{ii}(k). \tag{1}$$

Here, ω_0 is the frequency of the laser field, E_0 is its amplitude, Z is the mean ion charge state, $S_{ii}(k)$ is the ionic structure factor, $\omega_p^2 = 4\pi n_e e^2/m_e$ is the plasma frequency, $v_0 = eE_0/m_e\omega_0$ is the quiver velocity of a free electron moving in the laser field and J_n are Bessel functions of the first kind. The abbreviation $\boldsymbol{\epsilon} = -e\boldsymbol{E}_0/m_e\omega_0^2$ denotes the amplitude of the free electron oscillation. Expression (1) describes the collective electron–ion interaction screened by the classical dielectric function

$$\varepsilon(k,\omega) = 1 + \frac{\omega_p^2}{k^2}\lim_{\varepsilon\to 0}\int \mathrm{d}^3\boldsymbol{u}\frac{\boldsymbol{k}\cdot(\partial f_0/\partial\boldsymbol{u})}{\omega - \boldsymbol{k}\cdot\boldsymbol{u} + \mathrm{i}\varepsilon}. \tag{2}$$

Interestingly, the form (1) follows also from quantum theories (Vlassov and in Born approximation) [10, 11]. In this case, the dielectric response must be calculated within the quantum random phase approximation (RPA) [2].

Considering mainly short range collisions with $k > k_D$ ($k_D = \lambda_D^{-1} = \omega_p/v_{th}$ is the inverse Debye length and $v_{th} = k_B T_e/m_e$ is the thermal electron speed), Decker *et al* used a small argument approximation of the dielectric function [6]. This leads to the following approximate form of expression (1)

$$\frac{\nu_{ei}}{\omega_p} = \frac{Z}{(2\pi)^{3/2}}\frac{\omega_0^2 v_{th}^2}{\omega_p^2 v_0^2}\sum_{n=1}^{\infty} n^2\int_{-1}^{1}\mathrm{d}\mu\int_0^{\eta_{\max}}\mathrm{d}\eta\frac{J_n^2(\eta\mu k_D\epsilon)}{\eta^3}\exp\left(-\frac{1}{2}\frac{n^2\omega_0^2}{\eta^2\omega_p^2}\right). \tag{3}$$

Here, integration is limited to $\eta_{\max} = b_{\min}/b_{\max}$ (see below for a discussion of the cutoffs) and the ions are assumed to be uncorrelated: $S_{ii}(k) = 1$.

Often the weak field limit of expression (1) is used for its simplicity. In the limit of $v_0/v_{th} \ll 1$ and $\omega_0 \gg \omega_p$, one can approximate the integral and obtains

$$\frac{\nu_{ei}}{\omega_p} \approx \frac{Z}{3}\frac{1}{(2\pi)^{3/2}}\left(\frac{1}{n_e\lambda_{\mathrm{D}}^3}\right)\left[\ln\left(\sqrt{2}\frac{\omega_p}{\omega_0}\right) + \ln\Lambda\right]. \tag{4}$$

$\ln\Lambda = \ln(b_{\max}/b_{\min})$ is the (Spitzer-like) Coulomb logarithm with cutoffs usually set to be the electron Debye length and the distance of closest approach, i.e., $b_{\max} = \lambda_{\mathrm{D}}$ and $b_{\min} = \rho_\perp = Ze^2/m_e v_{th}^2$, respectively. Quantum diffraction can be approximately

incorporated by the interpolation $b_{\min} = \left(\bar{\lambda}^2 + \rho_\perp^2\right)^{1/2}$, where $\bar{\lambda} = \hbar/2m_e v_{th}$ is the deBroglie wavelength. One should, however, keep in mind that these cutoffs try to describe three aspects: strong classical collisions at small k, weak, Born-like collisions at large k and the weighting by the appropriate electron distribution. In the following, the upper combination is labelled 'LS' for the usual Landau–Spitzer description.

Obviously, the LS approach fails when $b_{\max}/b_{\min}$ becomes smaller than unity which occurs for dense, strongly coupled plasmas. This is partially linked to the use of straight electron trajectories when describing electron–ion collisions. Using hyperbolic orbits appropriate for Coulomb collisions, one obtains a Coulomb logarithm of the form $\ln\Lambda = \frac{1}{2}\ln\left(1 + b_{\max}^2/b_{\min}^2\right)$ [17–19]. This form will be labelled 'HLS' and uses $b_{\max}$ and $b_{\min}$ as defined in the LS description. It has the advantage of avoiding a total break-down, but the results become questionable at strong coupling, too.

Of course, a rigorous quantum-mechanical treatment of collisional absorption automatically avoids divergent integrals and, therefore, the problems related to the Coulomb logarithm vanish as well. Kremp *et al* [9] first derived a quantum kinetic equation for dense plasmas in strong laser fields where all scattering processes can be taken into account by appropriate generalised scattering rates [9]. Based on this kinetic approach, a quantum statistical description for the collision frequency ν_{ei} was derived [10]. The results are however limited to weak electron–ion interactions due to the use of the first Born approximation in the scattering rates. Similar results were obtain by applying Zubarev's linear response formalism to collisional absorption [13]. Here, strong collisions in weak fields can be included as well [14].

3. Comparison of the different approaches for the collision frequency

Results obtained from the different theories presented above are shown in figure 1. One finds good agreement for weakly coupled plasmas and increasing discrepancies for higher coupling strengths. Clearly, the models using LS-like Coulomb logarithms break down around $\Gamma = 1$. This fatal behaviour is avoided by considering hyperbolic orbits which can be regarded as absolutely necessary to allow for an easy-to-use classical description in a wider parameter space. For strongly coupled plasmas with $\Gamma > 1$, the HLS description gives smaller collision frequencies than quantum theories. This might be explained by the fact that the latter employ a Born approximation which becomes questionable in this region and tends to give larger scattering rates [17, 20]. A full description of collisional absorption should of course include a quantum description of strong electron–ion scattering.

The agreement of the results obtained by equations (4) and (3) underlines that the weak field approximation is applicable for the case with $v_0 = 0.2v_{th}$. The small differences are highlighted around the point where the LS Coulomb logarithm starts to fail.

A large body of published data exists for $n_e = 10^{22}$ cm^{-3}, $\omega_0 = 3\omega_p$ and $v_0 = 0.2v_{th}$. Here, most simulations and quantum theories agree within a factor of a few; even for strongly coupled plasmas. Differences may be traced back to the use of the Born approximation, the high-frequency approximation or the application of not well-defined quantum potentials in the classical MD simulations.

In conclusion, our results indicate that the field effects on the collisions are weak for the parameters considered. The application of a quantum dielectric function in equation (1) as well as the HLS Coulomb logarithm in the reduced model of equation (4) yield well-defined collision frequencies. Both approaches agree up to $\Gamma \approx 1$ and show the well-known deviations for higher coupling. Cutoffs fitted to T-matrix results of temperature relaxation rates [17] do not significantly modify the HLS results (not shown).

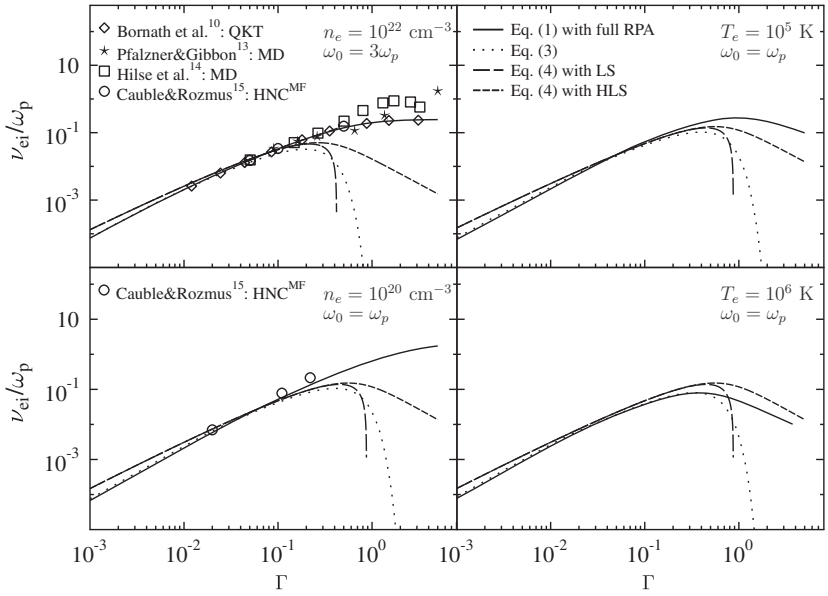

Figure 1. Comparison of different theoretical models and simulations for the collision frequency. Shown are results for constant densities (left) and temperatures (right) versus coupling strength of the plasma for a constant ratio of $v_0/v_{th} = 0.2$. HNC^{MF} denotes hypernetted chain calculations used in the memory function approach [16] and QKT stands for the quantum kinetic theory [10].

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